Phenomenology of neutrino physics in the Kaluza-Klein theories of low scale gravity

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We discuss the phenomenological consequences of theories which describe sterile neutrinos in large extra dimensions. We show that the Kaluza-Klein tower of the singlet neutrinos, albeit tiny individual contribution in electroweak processes, act cumulatively, giving rise to non-universality of the weak interactions of the light neutrinos and to flavour-violating radiative processes. Owing to these non-decoupling effects of th Kaluza–Klein neutrinos, we derive strong constraints on the parameters of the theory that originates from the non-observation of flavour-violating and universality-breaking phenomena. In this theory we propose a four-neutrino model which can reconcile the existing data coming from underground experiments in terms of neutrino oscillations, together with the hint from the LSND experiment and a possible neutrino contribution to the hot dark matter of the Universe.

Recently, it has been proposed [1] the radical possibility that the observed small value of the gravitational constat at long distance is ascribed to the spreading of the gravitational force in δ large extra spatial dimensions. It follows that the Standard Model (SM) fields are confined to a 3-brane configuration, while the large compactified dimensions are probed only by gravity and singlet, under SM gauge group, fields.

In this presentation we examine the phenomenological consequences of higher - dimensional isosinglet neutrinos on collider and lower energy experiments.

For our phenomenological study we adopt a variant [2] of the model discussed in Ref. [3] [4]. For definiteness, we consider a model that minimally extends the SM-field content by one singlet Dirac neutrino, $N(x,y)$, which propagates in a $[1+(3+\delta)]$-dimensional Minkowski space. The y-coordinates are compactified on a circle of radius R by applying the periodic identification: $y \equiv y + 2\pi R$.

The relevant part of the Lagrangian of our minimal model reads

$$\int_0^{2\pi R} d\vec{y}\ \Big[\ \bar{N}\Big(i\gamma^\mu\partial_\mu\ +\ i\gamma_{\vec{y}}\partial_{\vec{y}} - m\Big)N$$
$$+\ \delta(\vec{y})\ \Big(\sum_{l=e,\mu,\tau}\frac{h_l}{M_F^{\delta/2}}L_l\tilde{\Phi}\xi\ +\ \text{H.c.}\Big)\Big], \qquad (1)$$

where $\tilde{\Phi}$ ($<\tilde{\Phi}>= v = 174$Gev) and L_l are higgs and lepton doublets, respectively. We have assumed that only one two-component spinor ξ from the high dimensional spinor N couples to our 3 brane.

We can now express the two-component spinor ξ and its dirac partner η from high dimensional spinor N in terms of a Fourier series expansion as follows:

$$[\xi(x,y),\ \eta(x,y)] = \qquad (2)$$
$$\frac{1}{(2\pi R)^{\delta/2}}\ \sum_{\vec{n}}\ [\xi_{\vec{n}}(x),\ \eta_{\vec{n}}(x)]\ \exp\left(\frac{i\vec{n}\vec{y}}{R}\right).$$

Substituting into the Eq. (1) and then performing the $\vec{y}$ integration yields

$$\sum_{\vec{n}}\Bigg\{\bar{\xi}_{\vec{n}}(i\bar{\sigma}^\mu\partial_\mu)\xi_{\vec{n}} + \bar{\eta}_{\vec{n}}(i\bar{\sigma}^\mu\partial_\mu)\eta_{\vec{n}} - \qquad (3)$$
$$\Big[\xi_{\vec{n}}\Big((m+\frac{i|\vec{n}|}{R})\eta_{-\vec{n}} + \sum_{l=e,\mu,\tau}\bar{h}_l L_l\tilde{\Phi}\Big) + \text{H.c.}\Big]\Bigg\}$$

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where $\bar{h}_l = h_l \frac{M_F}{M_{\rm P}}$, $|\vec{n}|^2 \equiv \sum_{i=1}^{\delta} n_i^2$.

As it is shown in Eq. (3), there are small mixings mass terms, $\bar{h}_l v$, between any KK state $\eta_{\vec{n}}$ and left neutrinos. It was firstly noticed in [3,4], the four-dimensional Yukawa couplings $\bar{h}_l$ are naturally suppressed by the volume factor $M_F/M_{\rm P}$.

After diagonalizing the mass matrix these terms give rise to the small admixture of the left neutrinos to the heavy KK states, $\eta'_{\vec{n}}$, with the mass $\sqrt{m^2+\vec{n}^2/R^2}$

$$\eta'_{\vec{n}} \approx \eta_{\vec{n}} + B_{l,\vec{n}}\nu_L^l,\ B_{l,\vec{n}} \approx h_l \frac{vM_F}{M_P\sqrt{m^2+\vec{n}^2/R^2}}. \quad (4)$$

Applaying to the unitarity condition we arrive to the following light states

$$\nu^l_{light} \approx \frac{1}{\sqrt{1+\sum_{\vec{n}}|B_{l,\vec{n}}|^2}}(\nu_L^l + \sum_{\vec{n}} B_{l,\vec{n}}\eta'_{\vec{n}}). \quad (5)$$

The most striking feature of the higher-dimensional scenario, as it follows from Eq.(5), is the loss of the lepton universality in the electroweak processes.

It is useful to define the mixing parameters

$$(s_L^{\nu_l})^2 \equiv \sum_{\vec{n}} |B_{l,\vec{n}}|^2 \approx h_l^2 \frac{v^2}{M_F^2} \sum_{\vec{n}} \frac{M_F^4 M_P^{-2}}{(m^2+\frac{\vec{n}^2}{R^2})}$$
$$\approx \begin{cases} \dfrac{\pi h_l^2 v^2}{M_F^2} \ln\left(\dfrac{M_F^2}{m^2}+1\right), \delta = 2 \\[2ex] \dfrac{S_\delta}{\delta-2}\,\dfrac{h_l^2 v^2}{M_F^2}, \delta > 2\,. \end{cases} \quad (6)$$

In order to evaluate the sumation in the above equation we approximate the sum over the KK states by an integral, which has an upper ultraviolet (UV) cutoff at $M_F R$, above which string-threshold effects are expected to become more important.

As can be seen from Table 1, the mixings $(s_L^{\nu_l})^2$ may be constrained by a number of new-physics observables induced at the tree level. These observables measure possible non-universality effects in μ, τ and π decays. In this respect, in Table 1 we have defined $R_\pi = \Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$ and $R_{\tau\mu} = B(\tau\to e\nu\nu)/B(\tau\to\mu\nu\nu)$.

Owing to the tower of the KK singlet neutrinos which acts cumulatively in the loops, significant universality-breaking as well as flavour-violating effects are induced in electroweak processes involving γ-[5] and Z- boson [6] interactions. In particular, as has been explicitly shown recently,[6] we find that the cumulative presence of the KK states leads to an effective theory whose Yukawa interactions are mediated by order-unity Yukawa couplings of the original Lagrangian before compactification. In this case, we expect a higher-dimensional non-decoupling phenomenon analogous to the one studied earlier in renormalizable 4-dimensional theories.[7,8] For example, the effective lepton-flavour-violating vertex Zll' that occurs in $\mu\to eee$ exhibits the dependence: $\mathcal{T}(Zl'l) \ \propto\ h_l h_{l'}\,(v^2/M_F^2)\,\sum_{k=e,\mu,\tau}(h_k^2 v^2)/M_W^2$, i.e. its strength increases with the fourth power of the higher-dimensional Yukawa couplings. This should be contrasted with the respective photonic amplitude $\mathcal{T}(\gamma l'l) \propto h_l h_{l'}\, v^2/M_F^2$, whose strength increases only quadratically.

Based on this cumulative non-decoupling effect, we are able to derive strong limits on the M_F/h^2, for $h \geq 1$. As is displayed in Table 2, the strongest limits are obtained from $\mu \not\to eee$ and the absence of μ-to-e conversion in nuclei. Of course, the limits presented here contain some degree of uncertainty, which is inherent in all effective non-renormalizable theories with a cut-off scale, such as M_F. Nevertheless, our results are very useful, since they indicate the generic size of constraints that one has to encounter in model-building considerations with sterile neutrinos.[9,10]

Now we concentrate in the minimum brane-inspired scheme in which all elements required to explain the neutrino anomalies (the LSND/HDM as well as the solar and atmospheric mass scales) are generated by the physics of extra dimensions [10]. For definiteness, we will be considering a model that minimally extends the standard field content by one bulk neutrino, $N(x,y)$ (with zero high dimensional mass term), singlet under the SM gauge group. This propagates on a $[1+(3+n)]$-dimensional Minkowski space with $\delta \geq n$.

After integrating out heavy Kaluza-Klein states, the effective Lagrangian has the following

Table 1
Tree level limits on M_F/TeV from non-universal couplings of the neutrinos to W and Z bosons.

Observable	$h_e = h_\mu = h_\tau = h$ Lower limit on M_F/h $\delta = 2$	$\delta > 2$	$h_\mu = 0$ and $h_e = h_\tau$ Lower limit on M_F/h_τ $\delta = 2$	$\delta > 2$
$1 - \dfrac{\Gamma(\mu \to e\nu\nu)}{\Gamma_{\rm SM}(\mu \to e\nu\nu)}$	$8.9 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{3.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$6.3 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{\Gamma(Z \to \nu\nu)}{\Gamma_{\rm SM}(Z \to \nu\nu)}$	$5.9 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.4\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$4.8 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{1.9\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_\pi}{R_\pi^{\rm SM}}$	–	–	$18.7 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{7.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_{\tau\mu}}{R_{\tau\mu}^{\rm SM}}$	–	–	$5.7 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.3\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$

Table 2
One-loop-level limits on M_F/h^2.

Observable	$h_e = h_\mu = h_\tau = h \geq 1$ Lower limit on M_F/h^2 [TeV] $\delta = 2$	$\delta = 3$	$\delta = 6$
$\mathrm{Br}(\mu \to e\gamma)$	75	43	33
$\mathrm{Br}(\mu \to eee)$	250	230	200
$\mathrm{Br}(\mu\ {}^{48}_{22}\mathrm{Ti} \to e\ {}^{48}_{22}\mathrm{Ti})$	380	320	300

form

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \Big(\sum_{l=e,\mu,\tau} \bar{h}_l\, L_l \tilde{\Phi} \xi_0 + \text{H.c.} \Big), \quad (7)$$

where ξ_0 is the zero mode of the Kaluza-Klein states and now

$$\bar{h}_l = \left(\frac{M_F}{M_{\rm P}}\right)^{\frac{n}{\delta}} h_l. \quad (8)$$

In order to account for the LSND or hot dark matter mass scale $m_\nu \sim 1$ eV or so, we choose $\delta = 6$ and $n = 4$, giving $\bar{h}_l \sim 10^{-10} h_l$.

Now we turn to the Majorana masses for neutrinos. These are crucial in order to generate the mass splittings required in neutrino oscillation interpretations of the solar and atmospheric neutrino anomalies found in underground experiments. As shown in Ref. [3] the neutrinos may get small Majorana masses via interactions with distant branes where fermion number is maximally broken.

The Majorana part of the neutrino masses is then expected to be

$$m_{ll'} \sim f_{ll'} \frac{v^2}{M_F} \left(\frac{M_F}{M_{\rm P}}\right)^{2n/\delta - 4/\delta} \quad (9)$$

The neutrino mass matrix takes in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_s)$ $(\nu_s = \xi_0)$ the form

$$\mathcal{M}_\nu = \begin{pmatrix} m_{ll'} & M_l \\ M_{l'}^T & 0 \end{pmatrix}. \quad (10)$$

Here $M_l = \bar{h}_l v$. Note that the m_s entry in eq. 10 has been omitted since the bulk sector where the sterile neutrino ν_s lives is eight–dimensional.

In the limit that Dirac mass terms (M_l) are much bigger than Majorana mass terms $(m_{ll'})$, two of the neutrinos are massless and other two form Dirac state with a mass

$$m_\nu \equiv m_{LSND/HDM} = \sqrt{M_e^2 + M_\mu^2 + M_\tau^2} \quad (11)$$

This state is identified by two angles θ and φ defined as

$$\sin\theta = \frac{M_e}{m_\nu}, \quad \tan\varphi = \frac{M_\mu}{M_\tau}. \tag{12}$$

The entries $m_{ll'}$ only arise due to the breaking lepton number on distant branes. In the case $\delta = 6$ and $n = 4$ they are suppressed compared to the Dirac mass terms by the factor $\frac{v}{M_F}\frac{f_{ll'}}{h_l}$. These terms give masses to the lowest-lying neutrinos and also responsible for splitting Dirac state to two Majorana states. For suitable values of the parameters, these are in the right range to have a solution for solar and atmospheric neutrino deficit. More especifically, from the latest fits one needs [11]

$$\Delta m^2_{atm} \simeq 3.5 \times 10^{-3} eV^2 \tag{13}$$

in order to account for the full set of atmospheric neutrino data.

On the other hand the latest global analysis of solar neutrino data characterized by the best-fit points [11]

$$\Delta m^2_{LMA} \simeq 3.6 \times 10^{-5} eV^2 \tag{14}$$

$$\Delta m^2_{SMA} \simeq 5 \times 10^{-6} eV^2 \tag{15}$$

Now assuming naturalness, namely, that masses and splittings are of the same order $\Delta m_{atm} \simeq m_{ll'} \simeq \sqrt{\Delta m^2_\odot}$ and since $\Delta m^2_{atm} \simeq 2 m_\nu \Delta m_{atm}$, one finds

$$m_\nu \simeq 0.8 eV \text{for LMA} \tag{16}$$

$$m_\nu \simeq 0.3 eV \text{for SMA} \tag{17}$$

characterizing the order of magnitude of the LSND/HDM scale in the LMA/SMA cases.

Of course since clearly the solar mass splitting need not coincide exactly with the lightest state masses, the above estimates are meant to be crude order-of-magnitude estimates only. As a result the LSND/HDM scales both in the LMA and in the SMA case can be larger than estimated above.

If we assume that muonic neutrino coupling to the high dimensional spinor is dominant ($h_e, h_\tau \ll h_\mu \simeq 0.1$) the light sterile neutrino, ν_s, combines with ν_μ and form a Quasi–Dirac state, crucial to account for the hint coming from the LSND experiment, and may also contribute to the hot dark matter of the Universe. Apart from the mass of the Quasi–Dirac state [12], one has the splittings between its components, as well as the masses of two light active states. The splittings between the heavy states and that characterizing the lighter neutrinos will be associated with the explanations of the atmospheric and solar neutrino anomalies, respectively. The atmospheric neutrino deficit is ascribed to the ν_μ to ν_s oscilations. The solar neutrino problem could be solved via MSW small or large angle ν_e to ν_τ solutions. This reproduces exactly the phenomenological features of the model proposed in ref. [13], providing a complete scenario for the present neutrino anomalies.

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